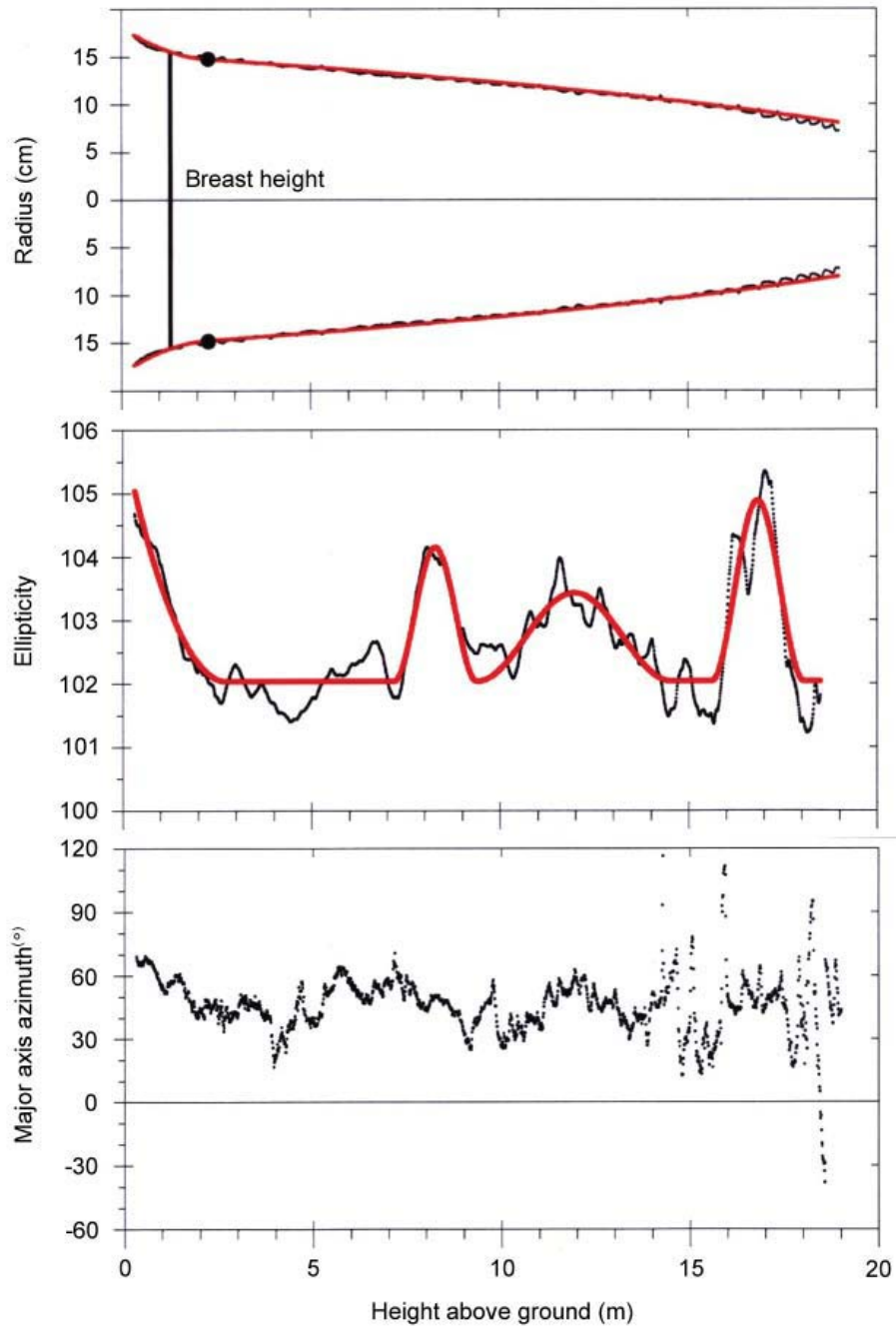


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Models of stem taper and cross-sectional eccentricity for Norway spruce and Scots pine

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Abstract

Tree models were developed whereby stem taper and cross-sectional eccentricity of Scots pine and Norway spruce stems could be predicted using site, stand and tree variables. The study was based on data from 246 Scots pine and 120 Norway spruce trees obtained through CT-scanning at 1–4 cm intervals along the stems. The intention was to develop models which could be applied, using these tree descriptors, to simulate external stem shape for sawmill conversion simulation studies. The following independent, descriptor variables were used in the models: Tree age, crown length, crown ratio, diameter at breast height (DBH), relative DBH, height to the lowest living branch, total height, actual height, height increment, site index, and temperature sum. Stem taper was modelled in a segmented function. Ellipticity, the quotient between the maximum and minimum diameter of an ellipse, was modelled in terms of a random coefficients, segmented function. The azimuth of the ellipse major axis was modelled in a linear function assuming a first-order autoregressive error structure.

Introduction

In order to generate data compatible with sawmill conversion simulation systems (e.g. Björklund & Julin 1998, Grundberg & Grönlund 1999, Nordmark 2002), detailed three-dimensional data of the stem shape is required. In some applications, such as forest management planning routines based on inventory data of a growing stock, this information is not readily available. However, the longitudinal variation of stem diameter can be generated through stem taper models, such as the well-known segmented polynomial equation which was first developed by Max and Burkhart (1976), and later used by many others for a large number of species (e.g. Demerchalk & Kozak 1977, Cao et al. 1980, Petersson 1999, and Sharma & Burkhart 2003). Many other model forms have been used for stem taper, but in an evaluation of different forms, Spångberg et al. (2001) found that the segmented polynomial equation was superior to those suggested by Edgren & Nylinder (1949) and Newnham (1992) for Scots pine (*Pinus sylvestris* L.) and Norway spruce (*Picea abies* [L.] Karst).

Cross-sectional shape could be assumed to be circular around centered pith, but this is a simplification that would result in over-estimation of the lumber volume recovery (and a corresponding under-estimation of the chip volume recovery) in a conversion simulation. Skatter & Hibe (1998) compared several models of cross-sectional shape and found that an ellipse performed adequately for Scots pine and Norway spruce. A Fourier coefficient model performed better, but required many parameter estimates.

The objectives of this study were to develop models which describe the external stem shape through longitudinal taper and cross-sectional elliptic models for Norway spruce and Scots pine, taking between-stand, -tree, and within-tree variation into account. The intention has been to develop a system of models which can be used together to generate the detailed data required by sawmill conversion simulation systems based on tree and stand variables.

MATERIALS AND METHODS

The study was based on 246 Scots pine trees sampled from 41 stands and 120 Norway spruce trees from 20 stands. The stands were chosen in order to get a broad distribution of altitude, latitude, site index, regeneration method, and thinning strategy for Swedish conditions. In each stand, the stems were divided into three DBH-classes around the stand quadratic mean DBH, with class limits at half a standard deviation above and below this mean. From each DBH-class, two stems were randomly chosen. Stand and tree properties have been summarized in Table 1.

Table 1.
Site, stand, tree and whorl data used in the study (S.D. = Standard deviation).

Property	Units	Scots pine				Norway spruce			
		Min.	Mean	S.D.	Max.	Min.	Mean	S.D.	Max.
<i>AGE</i>	years	36	98	30.6	153	51	98	34	152
<i>ALT</i>	m	50	239	99	420	80	180	99	310
<i>CL</i>	m	4.10	9.31	2.09	14.9	7.7	15.0	4.1	28.1
<i>CR</i>		0.19	0.43	0.083	0.64	0.32	0.62	0.13	0.88
<i>DBH</i>	cm	12.6	26.2	7.88	47.6	18.2	29.1	7.2	44.2
<i>Hllb</i>	m	3.5	11.2	3.61	19.8	2.1	9.2	3.8	18.1
<i>Ht</i>	m	9.7	20.0	4.67	29.0	19.7	24.2	4.23	34.5
<i>H_z</i>	m	0.34	7.70	4.93	20.4	0.31	8.34	5.99	27.2
<i>SI</i>	m	16	22.3	3.6	28	16	28	7.1	36
Observations									
<i>Cross-</i>	N° tree ⁻¹	480	1 406	344	1 993	790	1 678	463	2 705
<i>sections</i>	N° stand ⁻¹	5 311	8 392	1 780	11 110	7 435	10 069	2 156	14 055

The sampled trees were felled, cut into logs and transported to a laboratory for CT-scanning (Siemens SOMATOM AR.T.). One scan was taken every cm for Norway spruce and every 4 cm for Scots pine. The pith was identified manually in the digital image, and through digital image analysis, the radius from the pith to the stem surface (under bark) were measured automatically in 360 directions around the circumference (Grundberg 1999, Oja 2000). Since the project was carried out in collaboration with the sawmill industry, only the saw log section of stems were studied (small-end diameter of logs was 12 cm or larger).

Model Development

A model form, originally developed by Max and Burkhart (1976), was used fit the stem taper data. It is a nonlinear, segmented model. This form is quite flexible, and allows for application of different variables to different parts of the tree. Different conditions were applied to minimize the number of parameters while not losing predictive qualities as evaluated by Mean Square Error (MSE) and coefficient of determination (R^2). The SAS procedure “NLIN” using the Gauss-Newton method was used to fit the models (SAS Institute Inc. 1999).

An ellipse was first fitted for each cross-section using 180 diameters section⁻¹, resulting in estimation of maximum and minimum diameter, as well as the azimuth of the major axis. Ellipticity was then calculated as the quotient between maximum and minimum diameter. Using a segmented approach, different model forms for the sections and number of sections, a model fit was obtained stem-by-stem using “NLMIXED” procedure. Sections of increased ellipticity were identified, and fitted in different segments (above a threshold of 0.5 vertical m). Various tree- and stand-level variables were used in a mixed model approach to predict these parameter estimates using the SAS procedure “MIXED” and the restricted maximum likelihood method. These sub-models were evaluated with Akaike’s Information Criterion and residual variance (see SAS Institute Inc. 1999). Attempts to obtain a global model for the whole dataset were unsuccessful due to convergence problems.

Results

STEM TAPER

The model form was a segmented, double-quadratic equation (parameter estimates are listed in Table 2), as follows:

Table 2.
Parameter estimates for stem taper (Eq. 1).

Parameter	Scots pine		Norway spruce	
	Estimate	St.Err.	Estimate	St.Err.
Dependent variable: $\frac{D_z^2}{DBH^2}$				
a_{11}	$-2.76 \cdot 10^{-1}$	$4.29 \cdot 10^{-3}$	$-3.77 \cdot 10^{-1}$	$1.49 \cdot 10^{-2}$
a_{12}	$1.18 \cdot 10^{-1}$	$3.12 \cdot 10^{-3}$	$4.65 \cdot 10^{-1}$	$1.22 \cdot 10^{-2}$
a_{13}	$4.83 \cdot 10^1$	$4.03 \cdot 10^{-1}$	$1.44 \cdot 10^2$	2.78
a_{14}	$-6.86 \cdot 10^{-1}$	$3.60 \cdot 10^{-2}$	-3.20	$1.44 \cdot 10^{-1}$
a_{15}	$8.61 \cdot 10^{-2}$	$2.96 \cdot 10^{-4}$	$5.49 \cdot 10^{-2}$	7.9310^{-4}
a_{16}	$2.62 \cdot 10^{-4}$	$8.64 \cdot 10^{-6}$	–	
a_{17}	–		$1.98 \cdot 10^{-2}$	$1.22 \cdot 10^{-3}$
R^2	$96.5 \cdot 10^{-2}$		$94.5 \cdot 10^{-2}$	
MSE	$1.65 \cdot 10^{-3}$		$3.88 \cdot 10^{-3}$	

$$\frac{D_z^2}{DBH^2} = a_1 \left(\frac{H_z}{Ht} - 1 \right) + \left(a_{11} + a_{12} \frac{DBH}{Ht} \right) \left(\frac{H_z^2}{Ht^2} - 1 \right) + a_2 (a_{13} + a_{14} CL) \left(a_{15} + a_{16} DBH + a_{17} CR - \frac{H_z}{Ht} \right)^2 \quad (1),$$

where,

$$a_1 = \left(1 - \left(a_{11} + a_{12} \frac{DBH}{Ht} \right) \right) \left(\left(\frac{1.3}{Ht} \right)^2 - 1 \right) - a_3 \frac{(a_{13} + a_{14} CL) \left(a_{15} + a_{16} DBH + a_{17} CR - \frac{1.3}{Ht} \right)^2}{\left(\frac{1.3}{Ht} - 1 \right)} \quad (1.1),$$

$$a_2 = \begin{cases} 1 & \frac{H_z}{Ht} \leq a_{15} + a_{16} DBH + a_{17} CR \\ 0 & \frac{H_z}{Ht} > a_{15} + a_{16} DBH + a_{17} CR \end{cases}, \quad (1.2),$$

$$a_3 = \begin{cases} 1 & \frac{1.3}{Ht} \leq a_{15} + a_{16} DBH + a_{17} CR \\ 0 & \frac{1.3}{Ht} > a_{15} + a_{16} DBH + a_{17} CR \end{cases}, \quad (1.3),$$

D_z = Diameter at height z (cm),

H_z = Distance from the ground to height z (m),

Ht = Total tree height (m),

DBH = Diameter at breast height (cm),

CL = Crown length (total height minus height to lowest live branch; m),

CR = Crown ratio (CL/Ht),

a_{11} , a_{12} , a_{13} , a_{14} , a_{15} , a_{16} , and a_{17} are estimated parameters.

Stem diameter decreased substantially above an initial butt-swell near the ground. The first segment ended at a point determined by a DBH- or crown ratio-dependent height. The slope of the lower segment was bounded to be continuous with its adjacent upper segment at the join point. Finally, at the top of the tree, diameter was set to zero.

Ellipticity

Ellipticity, the quotient between the minimum and maximum stem diameter of an ellipse function at a given height, was fitted in a segmented model with a random number and length of segments. Near the stem base, a quadratic term reflected an increased eccentricity which reduced to a constant level higher up. Above this followed a random number of sections with increased eccentricity, modelled in a cosine function, which were separated by constant sections (parameter estimates are listed in Table 3):

$$\begin{aligned}
ELL_z &= C + b_1 H_z + b_2 H_z^2 & H_z < h_q \\
ELL_z &= C + b_1 h_q + b_2 h_q^2 & h_q \leq H_z < h_q + LAG_1 \\
ELL_z &= C + b_1 h_q + b_2 h_q^2 + ECC_z & h_q + LAG_1 \leq H_z < h_q + LAG_1 + WL_1 \\
ELL_z &= C + b_1 h_q + b_2 h_q^2 & h_q + LAG_1 + WL_1 \leq H_z < h_q + LAG_1 + WL_1 + LAG_2 \\
&\vdots \\
ELL_z &= C + b_1 h_q + b_2 h_q^2 + ECC_z & h_q + \sum_{k=1}^x (LAG_k + WL_{k-1}) \leq H_z < h_q + \sum_{k=1}^x (LAG_k + WL_k) \\
ELL_z &= C + b_1 h_q + b_2 h_q^2 & h_q + \sum_{k=1}^x (LAG_k + WL_k) \leq H_z < h_q + \sum_{k=1}^x (LAG_{k+1} + WL_k)
\end{aligned} \tag{2}$$

where,

$$b_1 = -2 b_2 h_q$$

$$b_2 = (C - LEVEL) / h_q^2$$

$$ECC_z = AMP_k \left(1 + \cos \left(\frac{360}{WL_k} \left(H_z - \left(h_q + \sum_{k=1}^x (LAG_k + WL_{k-1}) \right) + 180 \right) \right) \right)$$

$$C = b_{10} + b_{11} T_{sum} + b_{12} DBH + \alpha_{ij} \tag{2.1}$$

$$\text{LOG}_{10}(h_q + \Delta) = b_{20} + \alpha_{ij} \tag{2.2}$$

$$LEVEL = b_{310} + b_{311} T_{sum} + \gamma_i + \alpha(\gamma)_{ij} \tag{2.3.1}$$

$$\text{LOG}_{10}(LEVEL + \Delta) = b_{320} + \gamma_i + \alpha(\gamma)_{ij} \tag{2.3.2}$$

$$AMP_k = b_{40} + b_{41} T_{sum} + b_{42} ALT + b_{43} DBH + \alpha_{ij} + \tau(\gamma \alpha)_{ijk} \tag{2.4}$$

$$\text{LOG}_{10}(LAG_k + \Delta) = b_{50} + b_{51} DBH + \tau_{ijk} \tag{2.5}$$

$$\text{LOG}_{10}(WL_k + \Delta) = b_{60} + b_{61} SI + \tau_{ijk} \tag{2.6}$$

T_{sum} = Temperature sum (above a threshold of 5°C), derived from altitude and latitude (Morén & Perttu 1994; °C days),

ALT = Altitude (m above sea level),

SI = Site index, dominant height at 100 years (m),

C = Intercept,

h_q = Length of the lowest quadratic section,

$LEVEL$ = Constant level of ellipticity,

ECC_z = Amount of increased stem ellipticity above the constant level at height z ,

AMP_k = Maximum level of increased ellipticity for section k ,

LAG_k = Length of section k with constant ellipticity,

WL_k = Length of section k with increased ellipticity,

γ_i = Random, between-stand effect,

$\alpha(\gamma)_{ij}$ = Random between-tree effect, nested within stands,

α_{ij} = Random between-tree effect, pooled over all stands,

$\tau(\gamma \alpha)_{ijk}$ = Random between-section effect, nested within stands and trees,

τ_{ijk} = Random between-section effect, pooled over all stands and trees,

$b_{10}, b_{11}, b_{12}, b_{20}, b_{310}, b_{311}, b_{320}, b_{40}, b_{41}, b_{42}, b_{43}, b_{50}, b_{51}, b_{60},$ and b_{61} are estimated parameters.

Table 3.
Parameter estimates for ellipticity (Eq. 2).

Parameter	Scots pine		Norway spruce	
	Estimate	St.Err.	Estimate	St.Err.
Eq. 2.1: Dependent variable: C				
b_{10}	$1.03 \cdot 10^2$	1.07	$1.08 \cdot 10^2$	1.23
b_{11}	$2.57 \cdot 10^{-3}$	$1.01 \cdot 10^{-3}$	–	
b_{12}	–		$-9.8 \cdot 10^{-2}$	$4.15 \cdot 10^{-2}$
α_{ij}	10.3		9.19	

Eq. 2.2: Dependent variable: $\text{LOG}_{10}(h_q + \Delta)$				
b_{20}	$4.26 \cdot 10^{-1}$	$3.14 \cdot 10^{-2}$	$1.52 \cdot 10^{-1}$	$4.41 \cdot 10^{-2}$
Δ	$2.00 \cdot 10^{-1}$		$2.00 \cdot 10^{-1}$	
α_{ij}	$1.90 \cdot 10^{-1}$		$2.08 \cdot 10^{-1}$	
Eq. 2.3.1: Dependent variable: $Level$				
b_{310}	$1.00 \cdot 10^2$	$5.03 \cdot 10^{-1}$	–	
b_{311}	$1.82 \cdot 10^{-3}$	$4.74 \cdot 10^{-4}$	–	
γ_i	$2.77 \cdot 10^{-1}$		–	
$\alpha(\gamma)_{ij}$	$6.42 \cdot 10^{-1}$		–	
Eq. 2.3.2: Dep. variable: $\text{LOG}_{10}(Level + \Delta)$				
b_{320}	–		$2.35 \cdot 10^{-2}$	$4.55 \cdot 10^{-2}$
Δ	–		$-1.01 \cdot 10^2$	
γ_i	–		$2.58 \cdot 10^{-2}$	
$\alpha(\gamma)_{ij}$	–		$7.60 \cdot 10^{-2}$	
Eq. 2.4: Dependent variable: AMP_k				
b_{40}	–		$5.79 \cdot 10^{-1}$	$1.49 \cdot 10^{-1}$
b_{41}	–		$3.00 \cdot 10^{-4}$	$1.30 \cdot 10^{-4}$
b_{42}	$1.22 \cdot 10^{-3}$	$4.38 \cdot 10^{-2}$	–	
b_{43}	$2.61 \cdot 10^{-3}$	$1.94 \cdot 10^{-4}$	–	
α_{ij}	$4.38 \cdot 10^{-2}$		$2.47 \cdot 10^{-2}$	
$\tau(\gamma \alpha)_{ijk}$	$5.08 \cdot 10^{-1}$		$2.31 \cdot 10^{-1}$	
Eq. 2.5: Dependent variable: $\text{LOG}_{10}(LAG_k + \Delta)$				
b_{50}	$-2.43 \cdot 10^{-1}$	$2.54 \cdot 10^{-2}$	$-5.49 \cdot 10^{-1}$	$1.43 \cdot 10^{-1}$
b_{51}	–		$1.03 \cdot 10^{-2}$	$4.71 \cdot 10^{-3}$
Δ	$1.00 \cdot 10^{-1}$		$1.00 \cdot 10^{-1}$	
τ_{ijk}	$3.93 \cdot 10^{-1}$		$4.09 \cdot 10^{-1}$	
Eq. 2.6: Dependent variable: $\text{LOG}_{10}(WL_k + \Delta)$				
b_{60}	$1.41 \cdot 10^{-1}$	$1.39 \cdot 10^{-2}$	$4.10 \cdot 10^{-1}$	$7.60 \cdot 10^{-2}$
b_{61}	–		$-8.14 \cdot 10^{-3}$	$2.63 \cdot 10^{-3}$
Δ	$1.00 \cdot 10^{-1}$		$1.00 \cdot 10^{-1}$	
τ_{ijk}	$1.24 \cdot 10^{-1}$		$1.21 \cdot 10^{-1}$	

Azimuth of main axis

The azimuthal direction of the main axis (i.e. that associated with maximum diameter) was modeled in a mixed, non-linear function. The residual variance was modeled separately as a function of ellipticity, and assuming a first-order autoregressive error structure (parameter estimates are listed in Table 4):

Table 4.

Parameter estimates for azimuth of the main axis.

Parameter	Scots pine		Norway spruce	
Eq. 3: Dependent variable: EA_z				
c_1	2.99	1.78	-	
c_2	$-3.39 \cdot 10^{-1}$	$1.21 \cdot 10^{-1}$	-	
α_{ij}	27.8		38.6	
Eq. 3.1: Dependent variable: C				
c_{10}	$9.59 \cdot 10^1$	5.99	$1.05 \cdot 10^2$	8.64
γ_i	$6.26 \cdot 10^2$		$1.04 \cdot 10^3$	
$\alpha(\gamma)_{ij}$	$3.36 \cdot 10^3$		$2.28 \cdot 10^3$	

Eq. 3.2: Dependent variable: $\varepsilon_{k(i,j)}$				
c_{21}	3.0			
c_{22}	72.0			
c_{23}	-89.6			
τ_{ijk}	1.00			
c_{30}	$-8.73 \cdot 10^{-1}$	$1.62 \cdot 10^{-2}$		
c_{31}	$-8.17 \cdot 10^{-2}$	$1.61 \cdot 10^{-2}$		
γ_i	$1.19 \cdot 10^{-4}$			
$\alpha(\gamma)_{ij}$	$8.59 \cdot 10^{-4}$			

$$EA_z = C + \left(H_z - \frac{Ht}{2}\right) (c_1 + c_2 ALT^{0.5} + \alpha_{ij}) + \varepsilon_{k(i,j)} \quad (3),$$

$$C = c_{10} + \gamma_i + \alpha(\gamma)_{ij}, \quad (3.1),$$

$$\varepsilon_{k(i,j)} = -AR_1 \varepsilon_{k-1} + \sqrt{1-AR_1^2} \left(c_{21} + c_{22} e^{c_{23} \left(\frac{ELL_z}{100} - 1 \right)} \right) \tau_{ijk}, \quad (3.2),$$

$$AR_1 = c_{30} + c_{31} DBH_{rel} + \gamma_i + \alpha(\gamma)_{ij}, \quad (3.3),$$

where

EA_z = Azimuthal direction of the main axis at height z ($^\circ$: 0 = East; 90 = North; 180 = West; 270 = South),

DBH_{rel} = Tree DBH divided by stand quadratic mean DBH ,

$c_1, c_2, c_{10}, c_{21}, c_{22}, c_{23}, c_{30}$, and c_{31} are estimated parameters.

Discussion

On the basis of non-destructive measurements, models of stem taper and cross-sectional eccentricity could be developed. Stem taper was described in a segmented polynomial equation. Cross-sectional eccentricity was described through an ellipse function, whereby ellipticity was estimated by a random-coefficient, segmented function and the azimuthal direction of the main axis was estimated by a linear mixed function with a circular-normal, autoregressive error structure. The models were specifically designed to provide detailed data of external stem geometry required by sawmill conversion simulations systems (e.g. Björklund & Julin 1998, Grundberg & Grönlund 1999, and Nordmark 2002). Such an application of the Scots pine models is described elsewhere (Moberg & Nordmark 2005).

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